Math 299
Games, puzzles, and problems

July 5, 2021

Subtraction game

The following is a two player game. There are 21 coins in a pile. A move consists of taking away 1,2, or 3 coins from the pile, and players alternate taking turns. The player who takes the last coin wins the game.

1. Play a few games to get a sense for the strategy.
2. Who has an advantage? The player who goes first, or the player who goes second? Is one player guaranteed to win if they make the right moves?
3. Describe the winning strategy.
4. Play the same game, but starting with 20 coins in the pile. Is the game different? As in, does the strategy change, or does going first/second become less of an advantage/disadvantage?
5. Play the same game, but starting with 17 coins in the pile. Is the game different? As in, does the strategy change, or does going first/second become less of an advantage/disadvantage?
6. Play the same game, still with 21 coins, but now removing 4 coins is an allowable move. How does the strategy change?
7. Experiment with the following variables: changing the starting number of coins, and the maximum number of coins allowed in a single move. Try to describe a strategy to win this game where the pile starts with N coins, and any number of coins up to M is allowed to be removed in one move.

Factorials

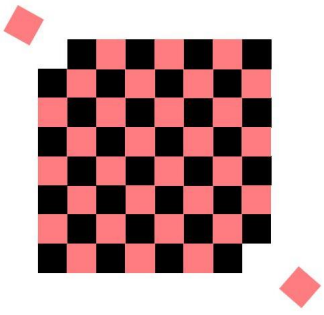
The **factorial** of a positive integer n is the product of all positive integers from 1 to n. We write $n!$ for "n factorial." For example, $5! = 1 * 2 * 3 * 4 * 5$.

1. The number 100! is much too big to calculate by hand or even with a calculator. The next few questions are about this number.
   (a) What digit is in the 1's place?
   (b) What digit is in the 10's place?
   (c) How many times is that same digit repeated at the end? (Is it also in the hundreds, thousands, etc. places?)
2. Answer question (c) above with 200! instead.
3. Answer question (c) above with 700! instead.

Chess board dominos

A chess board is an 8×8 grid of squares which alternate between white and black. Suppose you have a bag of dominos, and each domino is exactly the size of two chess squares.

1. Convince yourself that it is possible to cover the entire chess board using 32 dominos. This should not be hard.
2. Now consider a mutilated chess board, with two corner squares removed. There are 62 squares remaining. Is it possible to cover the chessboard with 31 dominos? If yes, provide an example. If not, justify why not.



Calendar number trick

Consult a typical calendar, with a box for each day and weeks of 7 days arranged with 1 week per row. Choose a 3×3 block of days anywhere in the calendar, which doesn't overlap multiple months.

1. Verify that the sum of the numbers (of the day of the month) of the 9 days in your block is equal to the center number of the block multiplied by 9. Prove that this is always true.

2. Is there a similar trick for a 4×4 block of days? 5×5? Generalize as much as possible.

Cutting pie

With one straight cut you can slice a pie into two pieces. A second cut that crosses the first one will produce four pieces, and a third cut can produce as many as seven pieces. What is the largest number of pieces that you can get with six straight cuts?